

## Droplet condensation in two-dimensional Bolgiano turbulence

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The evolution of microdroplets transported by a turbulent flow is considered. Droplets surrounded by moist air are able to grow by diffusion of water vapour on their surface. A two-dimensional model of dry Boussinesq thermal convection is considered in which the turbulent velocity field is driven by a temperature gradient. The evolution of the velocity, temperature, vapour fields and of droplet trajectories and radii is analysed by means of high-resolution direct numerical simulations. Despite the fact that the environment becomes drier and drier, a mean growth of droplets is obtained. The mechanism identified is based on the presence of correlations between the vapour field and droplet trajectories. Besides, a spreading of size distribution is observed, with the formation of droplets with very different sizes. Improvements with respect to previous models are discussed.

**Keywords:** Atmospheric turbulence; Turbulent convection; Direct numerical simulation; Two-dimensional turbulence

### 1. Introduction

The development of clouds is a topic of great interest, due to the clear importance of clouds for life on Earth. Their shape, lifetime, composition can alter important climate parameters. The numerous physical processes which take place inside these enormous natural laboratories, make the problem complex both from a theoretical and from an experimental point of view [1]; measures of small-scale characteristics in clouds are very difficult and their knowledge is still not complete. In this major problem some fundamental issues still must be fully understood. Measures in clouds reveal a broad size distribution of small droplets while classical air-parcel models point to narrowing size spectra during the condensation stage. Indeed, as long as the collisions are rare (for droplets smaller than about  $20\text{ }\mu\text{m}$  in radius), condensation is the only mechanism which can sustain droplet growth. However, the classical model of condensation inside a fluid parcel [2] predicts a narrowing of droplet size spectrum. Though some improvement has been achieved by including the effect of entrainment [3] of dry air inside the cloud, such a mechanism can only partially justify the presence of a wide range of droplet sizes, as it concerns only the cloud boundaries, whereas the spreading of size distribution is measured in adiabatic cloud cores as well. Other mechanisms have been proposed to explain such a property of the inner part of the cloud: the effect of stochastic fluctuations in the vapour field has been considered in [4], as well as in [5, 6] where droplet reaction on the vapour field is responsible for its local fluctuations. The effects of developed turbulence have been first

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included in [7]. There, an ascending fluid parcel of moist air was considered, where droplets were able to grow by condensation: this resulted in a very limited degree of spreading of the size distribution.

In a recent paper [8], a very simple model of condensation in clouds was considered. Such simplicity allowed the authors to isolate a spreading mechanism due to correlations produced by the carrying flow: the idea was that droplets and vapour are correlated since they are advected by the same velocity field. Therefore, every droplet experiences the same ambient conditions for a timescale comparable to the large-eddy turnover time. In this picture, droplets belonging to a very moist region are able to grow faster than other droplets correlated to less moist regions, thus spreading the size distribution. Finally, droplets correlated to dry regions will evaporate and disappear leaving dry regions void of particles. That shows the importance of large-scale spatial fluctuations of the vapour field: droplets behave differently since they grow within different conditions of humidity. Clearly, in order to observe this mechanism the investigation of a large part of the cloud is required.

The aim of the present work is to verify the robustness of the identified mechanism in a more realistic framework including previously neglected aspects. In particular, in [8] homogeneous isotropic turbulence was considered, sustained by an injection of energy modelled as a random process. Moreover, the back reaction of droplets on the vapour field was neglected, due to the small number of droplets. We wish now to make a further step by including two additional ingredients: the thermal convection due to an imposed temperature gradient and the effect of droplet feedback on vapour. Even if, in principle, results can substantially change, because of the different dynamics with the peculiar statistical properties of the Bolgiano regime for all the involved fields, we show the persistence of the described mechanism with a considerable broadening of size distribution. The qualitative picture does not present any remarkable difference.

## 2. The model

This section is devoted to describe the model we introduce to perform a numerical analysis of the condensation problem in two dimensions. As discussed above, we consider improvements of the model presented in [8] to describe more realistic features, namely the convection due to thermal unstable stratification and the feedback of droplets on the vapour field.

Convection is driven by the temperature field  $T$ , which is an active scalar coupled to the velocity field via the buoyancy term. In particular, we are dealing with a stratified medium (our idealised atmosphere), where a cooling contribution proportional to the vertical component  $w$  of the velocity field ( $w = \mathbf{v} \cdot \hat{\mathbf{z}}$ ) is explicitly taken into account in the equation for the temperature field. The physical meaning of such a term is the cooling of an ascending air parcel at a constant rate  $\Gamma_a$  (adiabatic lapse rate). Note that we set  $\Gamma_a$  to the moist value, which amounts to incorporating an average contribution due to latent heat effects. Because of stratification we can split the temperature as the sum of two terms, a mean profile  $-Gz$  and a fluctuation  $T'$  around it with zero average which satisfies the equation

$$\partial_t T' + \mathbf{v} \cdot \nabla T' = (G - \Gamma_a)w + \kappa \Delta T', \quad (1)$$

where the term  $(G - \Gamma_a)w$  is the injection of scalar fluctuations and  $\kappa$  is the molecular diffusivity. Note that, here, the role of temperature is to sustain, as a local pumping term, velocity fluctuations. The simplest, even if non-trivial, way to obtain a convective background

is to focus, as a first step, on a two-dimensional setup. This is the essence of the Bolgiano turbulent regime.

Temperature fluctuations drive the vorticity field  $\omega$ , which follows the equation

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \beta \mathbf{g} \times \nabla T' + \nu \Delta \omega, \quad (2)$$

where  $\mathbf{g}$  is the gravitational acceleration,  $\beta$  is the thermal expansion coefficient and  $\nu$  is the kinematic viscosity. Since compositional effects are generally thought to be small (see [9]), we consider here dry buoyancy. We focus on the *Bolgiano regime* [10] in which the dynamics is ruled by the scale-by-scale equilibrium between the inertial term  $\mathbf{v} \cdot \nabla \omega$  and the Boussinesq buoyancy term  $\beta \mathbf{g} \times \nabla T'$  (see the appendix for a description of this regime).

Let us now discuss the inclusion of the back reaction of droplets on the vapour field. To do that, we first introduce an important ingredient of the considered model: the vapour field. The *supersaturation* field  $s(\mathbf{x}, t)$  quantifies the amount of vapour which is present in the position  $\mathbf{x}$  at time  $t$ . It is defined by  $s(\mathbf{x}, t) \equiv e(\mathbf{x}, t)/e_s(\mathbf{x}, t) - 1$ , where  $e$  and  $e_s$  are the vapour pressure and the saturation vapour pressure, respectively. In the regions where  $s$  is positive (*moist regions*) vapour exceeds the equilibrium point and therefore tends to condense. Conversely, *dry regions* are poor of vapour and water here is expected to evaporate. According to a simple generalisation of the well-known Twomey's model [2], where we considered space/time dependences brought by the turbulent velocity field instead of the sole time dependence of the classical Twomey's model, the supersaturation field is considered as a passive scalar and is thus ruled by the equation

$$\partial_t s + \mathbf{v} \cdot \nabla s = A_1 w - \frac{1}{\tau_s} s + D \Delta s, \quad (3)$$

where  $A_1 w$  is a local source (or sink) of supersaturation due to the cooling (or heating) of an ascending (or descending) volume, while  $-s/\tau_s$  accounts for the *local* feedback of droplets on vapour. We wish to introduce such interaction at local level in order to describe the spatial variations in the statistics of  $s$ , which depend on how many droplets are locally present. The *absorption time*  $\tau_s$  is thus considered here as a field, evolving in space and time according to the size and number density of droplets. During evolution, in each point  $\mathbf{x}$ ,  $\tau_s$  is inversely proportional to the sum of the radii  $R_i$  of the  $N(\mathbf{x}, t)$  droplets present in a little volume around  $\mathbf{x}$  (see e.g. [11]):

$$\frac{1}{\tau_s}(\mathbf{x}, t) = A \sum_{i=1}^{N(\mathbf{x}, t)} R_i(t), \quad (4)$$

where  $A$  is a function of thermodynamic parameters and temperature. The coefficients  $A$  and  $A_1$  can be considered constant since their maximum variation within the whole domain (few hundreds meters) is less than 1% (see [12]). In the numerical procedure we consider that every droplet affects the value of  $\tau_s$  in the four nodes surrounding its position. The weight of the contribution to each node depends on the distance from the node via bilinear interpolation.

The Lagrangian equations for the trajectory and radius of each droplet are required to complete the model. Thanks to the small droplet Reynolds numbers, the small Stokes numbers and the small mass loading (ratio of the total mass of particles and the mass of the carrier fluid), we can consider droplets as independent, passive particles (see [13]). Moreover we assume droplets as tracer particles (see discussion in [8]), since the Stokes timescale of the largest droplet in our simulations is about 0.01 s, much shorter than the shortest timescale

associated with our turbulent velocity field ( $\tau_\eta \approx 2.5$  s). Hence, for the  $i$ th droplet we obtain

$$\frac{d}{dt} \mathbf{X}_i(t) = \mathbf{v}(\mathbf{X}_i(t), t) + \sqrt{2D} \boldsymbol{\eta}_i \quad (5)$$

$$\frac{d}{dt} R_i^2(t) = 2A_3 s(\mathbf{X}_i(t), t), \quad (6)$$

where  $\boldsymbol{\eta}_i$  are independent white noises and  $A_3$  can be considered as a constant. Note that gravity acceleration has been neglected in equation (5), since the terminal velocity is a small fraction (from 0.01 to 0.25) of the minimum velocity we can resolve.

The feedback of droplets on vapour has now an important role linked to the correlation effect shown in [8]. If such effect was still present, dry regions would be void of droplets, providing no vapour loss there. All droplets would be segregated in moist regions where they would grow consuming the surrounding vapour thus slowing down their growth. This could change the prediction on the spreading of size distribution. Moreover, let us stress that the air can become undersaturated on average, as discussed in the following; thus from equation (6), with a mean-field-type argument, we would not expect any mean growth of radii. The aim of the following section is therefore to quantify the importance of the droplet back reaction on vapour in the present framework.

### 3. Results

As discussed in the previous section, we focus on a regime which presents different statistical properties of velocity, temperature and supersaturation with respect to Kolmogorov 1941s. We wish now to consider the evolution of droplets advected by a convective velocity field and interacting with the described supersaturation field. We performed a series of high resolution ( $1024^2$ ) direct numerical simulations of model equations (1)–(3), (5) and (6), integrating the 2D velocity field  $\mathbf{v}$ , the temperature field  $T$  and the supersaturation field  $s$  by a standard 2/3-dealiased pseudospectral method on a doubly periodic square domain of length  $2\pi$ . The dissipative terms have been substituted, as customary, by hyperviscous terms of order eight for the viscosity and of order 4 for the diffusivities. A linear friction term is added to the vorticity equation to prevent the energy from accumulating at the lowest accessible modes. The time evolution is implemented by a standard second-order Runge–Kutta scheme with a time step of about  $\tau_\eta/250$ . Once the stationary state is reached (after about  $17 \tau_L$ ), we put one million droplets randomly in space and with the initial size around  $R_0 = 4 \mu\text{m}$ , growing according to equation (6); we follow their evolution for over  $2 \tau_L$ . Since the atmospheric Reynolds number can reach enormous values, around  $10^7$ – $10^8$ , we have to choose the range of scales we are interested in. If we focus the attention on a small fluid parcel, describing the small-scale dynamics in great detail, we cannot take into account the effect of large-scale fluctuations of the fields. Namely we would have almost the same ambient conditions for all droplets inside the computational domain, because fluctuations are tiny at small scales. But droplets transported by the turbulent flow are in fact able to span a very large volume of the cloud and therefore to experience very different ambient conditions. We then choose to simulate the evolution of the whole cloud, not resolving the small-scale details (see [8] for discussion).

Before showing the results, we give a remark on the mean vapour field. The equation of evolution of  $s$ , does not preserve its average: it is not clear *a priori* the general trend of the

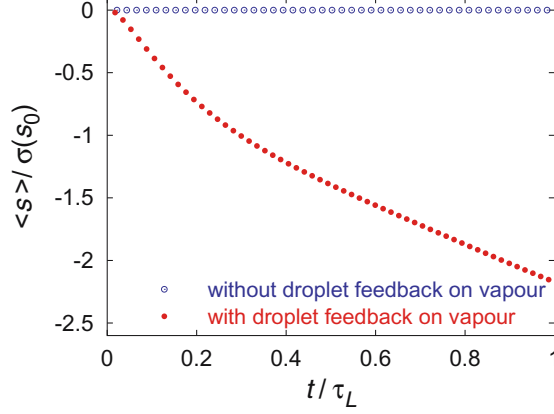


Figure 1. Mean supersaturation evolution in time. Here,  $\sigma(s_0)$  is the standard deviation of the initial field  $s(\mathbf{x}, 0)$  and  $\tau_L$  is the large-eddy turnover time. Note that there is a constant negative trend which suggests the presence of correlations between  $s$  and  $\tau_s^{-1}$  as discussed in the text.

mean supersaturation  $\langle s \rangle$ . Indeed, by averaging equation (3) we obtain

$$\frac{d}{dt} \langle s(\mathbf{x}, t) \rangle = - \left\langle \frac{s(\mathbf{x}, t)}{\tau_s(\mathbf{x}, t)} \right\rangle. \quad (7)$$

Let us guess the sign of  $\langle s/\tau_s \rangle$ . If the fields  $s(\mathbf{x}, t)$  and  $\tau_s^{-1}(\mathbf{x}, t)$  were independent,  $\langle s/\tau_s \rangle = \langle s \rangle \langle \tau_s^{-1} \rangle$  and the rhs of equation (7) would have the sign opposite to  $\langle s \rangle$ , given that  $\langle \tau_s^{-1} \rangle > 0$ . In this case equation (7) tells us that the mean supersaturation decays in time. Consider now the extremely correlated situation in which all droplets are concentrated in moist regions. In this case, dry regions would be totally void of droplets: here  $\tau_s^{-1}$  would vanish (see equation (4)). In contrast, we would have droplets—and therefore  $\tau_s^{-1} > 0$ —only in moist regions, i.e., where  $s > 0$ , providing  $\langle s\tau_s^{-1} \rangle > 0$ . From equation (7) we can conclude that in the presence of correlations there is a general negative trend of  $\langle s \rangle$ . The result shown in figure 1 demonstrates that the average supersaturation has a constant negative trend which confirms the presence of correlations between droplets and vapour. As a consequence we are dealing with a drier and drier environment, which works against the growth of droplets.

We wish now to ask directly whether correlations are actually present. The answer is given in figure 2, where we show the snapshots of the field  $s(\mathbf{x}, t)$  beside the field  $\tau_s^{-1}(\mathbf{x}, t)$ . The figure shows that droplets are present (i.e.  $\tau_s^{-1}(\mathbf{x}, t) > 0$  in the left panel) only in moist regions (shown in light blue in the right panel).

From these results we can conclude that the dynamical state produced by thermal convection does not affect the presence of correlation and the segregation of droplets in moist regions detected in the framework of homogeneous isotropic turbulence [8]. We come now to the quantitative description of droplet size spectrum evolution.

The picture we have qualitatively drawn for the back reaction of droplets on vapour is that every droplet grows absorbing the surrounding vapour, providing a drier and drier environment and slowing down its growth. Since  $\tau_s^{-1}$  is proportional to droplet radii, the larger the droplet, the faster the vapour loss. Therefore we expect that such feedback results in a slowing down of the mean droplet growth and of the spectrum broadening as well. In figure 3 we compare the mean radius growth obtained by two simulations beginning with the same initial condition: in order to obtain the effect of droplet feedback on water vapour, one of the two simulations neglects the term  $-s/\tau_s$  in equation (3). As we expected, neglecting the back reaction of droplets on vapour results in an overestimate of the mean growth rate. Indeed, in a large-eddy

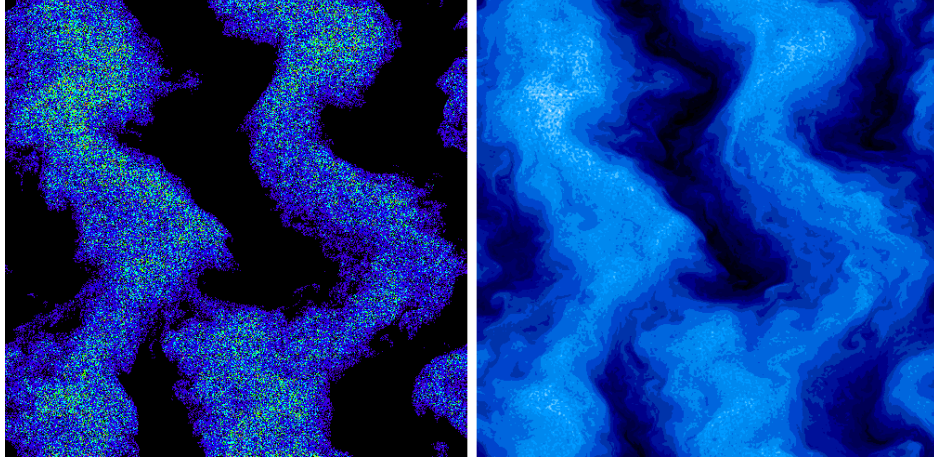


Figure 2. Left panel: snapshot of the field  $\tau_s^{-1}(x, t)$  obtained by summing the radius of droplets inside a small volume  $\delta V_x$  around point  $x$  at time  $t = 0.1 \tau_L$  (entire domain). From equation (4),  $\tau_s^{-1}$  vanishes where no particles are present (black regions). Right panel: snapshot of the supersaturation field  $s(x, t)$  at the same time  $t = 0.1 \tau_L$ . Dry regions are represented in dark blue, while moist ones are represented in light blue. Dark regions of the left panel correspond to dark regions of the right one: dry regions are void of droplets. Note that the different patterns with respect to figure 1 in [8] are due to the different dynamical regime considered.

turnover time  $\tau_L$ , which corresponds to  $\sim 100$  s, droplets grow due to the sole condensation mechanism, until  $\sim 50 \mu\text{m}$  on average. This value is actually much larger than observed in real clouds (see e.g. [1]). Taking into account this term goes in the direction of slowing down the condensational growth, making the prediction more realistic.

With the same spirit we compare the evolution of droplet size spectrum obtained with the same couple of simulations. The result of the comparison is shown in figure 4 for the two cases with and without the feedback of droplets on vapour: again the expectation of a slower spreading was right. However, the quantitative data show that also taking into account the droplet feedback on vapour, the condensation stage produces droplets with a significantly broad size distribution.

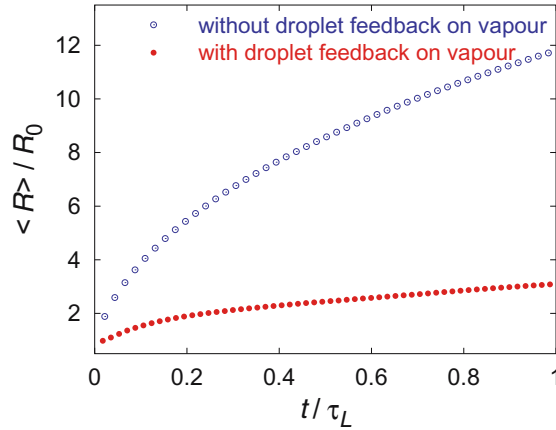


Figure 3. Growth of the mean radius in time, for the case with (red line) and without (blue line) the sink term  $-s/\tau_s$ . Note that neglecting the feedback of droplets on water vapour results in an overestimate of the mean growth of droplets.

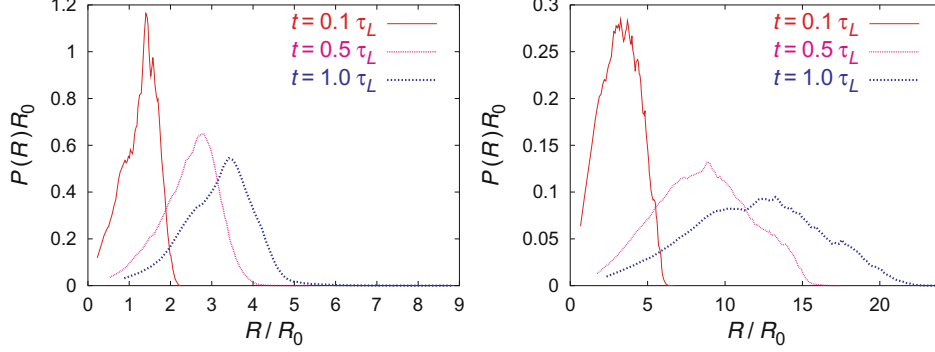


Figure 4. Droplet size spectrum for the case with accounted (left panel) and neglected (right panel) feedback of droplets on vapour. Note that, since  $R_0 = 4 \mu\text{m}$ , neglecting the feedback term results in the production of droplets with radii up to  $85\text{--}90 \mu\text{m}$  after one  $\tau_L$  ( $\approx 100$  s), while accounting for such term results in a more reasonable maximum size of  $20\text{--}22 \mu\text{m}$ .

#### 4. Conclusions and perspectives

The aim of the present work was to improve the results obtained in [8] for droplet condensation in highly idealised models of turbulent clouds. Two new ingredients were introduced to move towards a more realistic framework. We considered indeed thermal convection to drive the velocity field through buoyancy. The latter mechanism changes the stationary state of the turbulent system and yields a very different statistics with respect to the classical 2D inverse cascade regime. Moreover, we consider here the feedback of droplets on vapour which was previously neglected. Such interaction is modelled by an additional term in the equation of evolution for the vapour field and, heuristically, works against the fast growth and spreading of droplet radii. The results show that the spreading mechanism identified in [8] is still present despite the different statistical properties of the turbulent regime analysed. The slowing down of spreading and mean growth does not affect the general behaviour of the droplet size spectrum; in contrast, it results in a more realistic prediction.

A necessary step in the direction of a description of droplet condensation is a complete 3D simulation of the system. Such issue could support the significance of the mechanism identified for a more exhaustive picture of droplet growth by condensation in warm clouds.

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#### Appendix A. The Bolgiano regime

The two-dimensional Boussinesq turbulent convection is described by the following couple of partial differential equations [14]:

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = -\beta \nabla T \times \mathbf{g} + \nu \Delta \omega \quad (\text{A1})$$

$$\partial_t T + \mathbf{v} \cdot \nabla T = \kappa \Delta T. \quad (\text{A2})$$

Here,  $T$  is the temperature field and the vorticity  $\omega$  is a scalar field, since in a plane flow  $\nabla \times \mathbf{v}$  has only the normal component. In order to mimic both the fact that the fluid is supposed to be heated from below and that the mean temperature profile in a cloud is an almost linearly decreasing function of the elevation, a mean profile  $\langle T(\mathbf{r}, t) \rangle = \mathbf{G} \cdot \mathbf{r}$  is assumed, with a constant gradient  $\mathbf{G}$  pointing downwards in the direction of the gravity field. In a similar model, studied in [15–17], no mean gradient is present and a forcing term is added to the equation for the temperature field. There, the average temperature becomes constant and turbulence is not excited locally, but emitted from the underlying layer.

In equation (A2) the temperature field affects the vorticity through the buoyancy forces, thus providing an example of active scalar turbulence. At large enough values of  $\beta$ , the buoyancy forces can equilibrate the inertial terms in the velocity dynamics, while the temperature fluctuations cascade towards the small scales at a rate  $\epsilon$ .

Let us briefly recall the phenomenology of 2D turbulent convection (for the 3D case, see e.g. [18, 19]). The balance of buoyancy and inertial terms in equation (A1) introduces the Bolgiano length scale  $l_B$  [18]. At small scales,  $r \ll l_B$ , the inertial term is larger than buoyancy forces and the temperature is basically a passive scalar. At scales  $r \gg l_B$ , buoyancy dominates and affects the velocity, which performs an inverse energy transfer in two dimensions. However, unlike what happens in the usual 2D Navier–Stokes turbulence, the kinetic energy input rate here depends on the scale. Dimensional arguments yield:

$$\epsilon(r) = \beta \mathbf{g} \cdot \langle \mathbf{v}(\mathbf{x} + \mathbf{r}, t) T(\mathbf{x}, t) \rangle \sim r^{4/5} \quad (\text{A3})$$

and the dimensional expectations for both velocity and temperature structure functions:

$$S_N^v(r) = \langle (\delta_r v_{\parallel})^N \rangle \sim [\epsilon(r) r]^{N/3} \sim r^{\zeta_N^v} \quad \zeta_N^v = \frac{3N}{5} \quad (\text{A4})$$

$$S_N^T(r) = \langle (\delta_r T)^N \rangle \sim r^{\zeta_N^T} \quad \zeta_N^T = \frac{N}{5}. \quad (\text{A5})$$

Numerical results show that no intermittency corrections are observed for the velocity, whereas the temperature field appears strongly intermittent and saturates to a constant value (see [14]). Second-order structure functions for both velocity and temperature are reported in figure A1, in good agreement with the dimensional predictions (A4) and (A5).

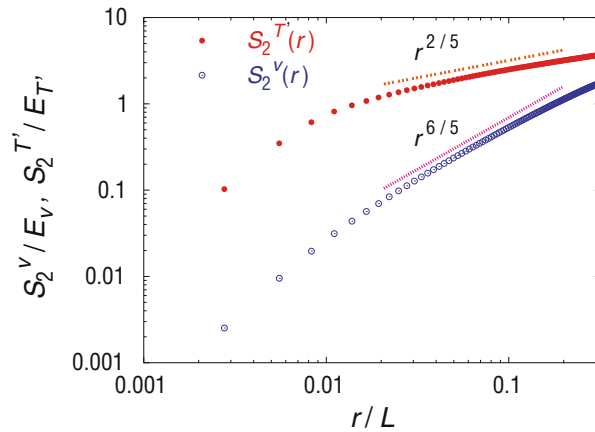


Figure A1. Second-order structure functions of both velocity (curve below) and thermic fluctuations (curve above) in the stationary state. Droplet evolution during the condensation stage has been studied in this state. Slopes are relative to the dimensional predictions (A4) and (A5). Ordinates are made dimensionless through  $E_T \equiv \langle T'^2/2 \rangle^{1/2}$  and  $E_v \equiv \langle v^2/2 \rangle^{1/2}$ .



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