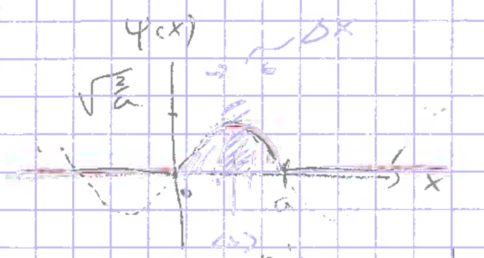
Ex 20

Valeurs moyennes et variance



Dimensione da a : [m]

$$\psi_{(x)} = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$$

$0 \leq x \leq a$; $\psi(x) = 0$ [illegible]

Normalisé?

- $$\langle \psi | \psi \rangle = \int_0^a \psi^*_{(x)} \psi_{(x)} = \frac{2}{a} \int_0^a \sin^2\left(\frac{\pi x}{a}\right) dx =$$

$$= \frac{2}{a} \left[\frac{1}{2} \frac{a}{\pi} \left(\frac{\pi x}{a} - \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi x}{a}\right) \right) \right]_0^a =$$

$$= \frac{1}{\pi} \left(\frac{\pi a}{a} - 0 - 0 - 0 \right) = 1 \quad !$$

- $$\langle x \rangle = \langle \psi | X | \psi \rangle = \int_{-\infty}^{\infty} \psi^*_{(x)} \, x \, \psi_{(x)} \, dx =$$

$$= \frac{2}{a} \int_0^a x \sin^2\left(\frac{\pi x}{a}\right) dx =$$

$$= \frac{2}{a} \left[\frac{x^2}{4} - \frac{a x}{4\pi} \sin\left(\frac{2\pi x}{a}\right) - \frac{a^2}{8\pi^2} \cos\left(\frac{2\pi x}{a}\right) \right]_0^a =$$

$$= \frac{2}{a} \left(\frac{a^2}{4} - \frac{a^2}{8\pi^2} + \frac{a^2}{8\pi^2} \right) = \frac{a}{2}$$

- $$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x^2 \rangle = \int_0^{\infty} \psi^*_{(x)} \, x^2 \, \psi_{(x)} \, dx = \frac{2}{a} \int_0^a x^2 \sin^2\left(\frac{\pi x}{a}\right) dx =$$

$$= \frac{2}{a} \frac{1}{24\pi^3} \left[3a \left(a^2 - 2\pi^2 x^2\right) \sin\left(\frac{2\pi x}{a}\right) - 6\pi a^2 x \cos\left(\frac{2\pi x}{a}\right) + 4\pi^3 x^3 \right]_0^a =$$

$$= \frac{1}{12 a \pi^3} \left(-6\pi a^3 + 4\pi^3 a^3 \right) = a^2 \left(\frac{1}{3} - \frac{1}{2\pi^2} \right)$$

$$\Rightarrow (\Delta x)^2 = a^2\left(\frac{1}{3} - \frac{1}{2\pi^2}\right) - \frac{a^2}{4} = a^2\left(\frac{1}{12} - \frac{1}{2\pi^2}\right)$$

$$\Delta x = a\sqrt{\frac{1}{12} - \frac{1}{2\pi^2}} \qquad (\simeq a \times 0.18)$$

- $$p = -i\hbar\frac{d}{dx} \qquad \left[\frac{m\,kg}{s}\right] \quad \left[\frac{Js}{m} = \frac{s\,Nm}{m} = \frac{s\,kg\,m}{s^2} = \frac{kg\,m}{s}\right]$$

$$\langle p \rangle = \int_0^a \psi^*_{(x)}\left(-i\hbar\frac{d}{dx}\right)\psi_{(x)}\,dx =$$

$$= -\frac{i\hbar 2}{a}\int_0^a \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi x}{a}\right)\cdot\frac{\pi}{a}\,dx =$$

$$= -\frac{2i\hbar\pi}{a^2}\left[-\frac{a}{4\pi}\cos\left(\frac{2\pi x}{a}\right)\right]_0^a = \frac{i\hbar}{2a}(1-1) = 0$$

- $$(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2 = \langle p^2 \rangle$$

$$\langle p^2 \rangle = \int_0^a \psi^*_{(x)}\left(-i\hbar\frac{d}{dx}\right)^2\psi_{(x)} =$$

$$= -\frac{2\hbar^2}{a}\int_0^a \sin\left(\frac{\pi x}{a}\right)\left\{-\sin\left(\frac{\pi x}{a}\right)\right\}\cdot\frac{\pi^2}{a^2}\,dx =$$

$$= \frac{2\pi^2\hbar^2}{a^3}\int_0^a \sin^2\left(\frac{\pi x}{a}\right)dx = \frac{2\pi^2\hbar^2}{a^3}\left[\frac{x}{2} - \frac{a}{4\pi}\sin\left(\frac{2\pi x}{a}\right)\right]_0^a =$$

$$= \frac{\pi^2\hbar^2}{a^2}$$

$$\Delta p = \frac{\pi\hbar}{a} \qquad \left(\text{Pot. ext.} \Rightarrow k = \frac{\pi}{a} \Rightarrow \Delta p = \hbar k = \frac{\pi\hbar}{a}\right)$$

- $$\Delta p\,\Delta x = \pi\hbar\sqrt{\frac{1}{12} - \frac{1}{2\pi^2}} = \hbar\sqrt{\frac{\pi^2}{12} - \frac{1}{2}} =$$

$$= \hbar \cdot 0{,}568 > \frac{\hbar}{2}$$