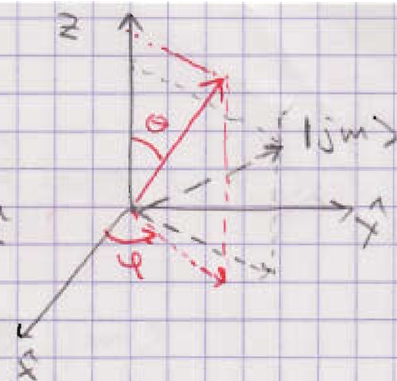
(L3) (42)

~~Exercice~~ 39

Rotation d'un moment angulaire



$R(\varphi)_{\hat{z}} = \exp\left\{-\frac{i}{\cancel{\hbar}}\varphi J_z\right\}$: rotation d'angle $\varphi / O\hat{z}$

$R(\theta)_{\hat{y}} = \exp\left\{-\frac{i}{\hbar}\theta J_y\right\}$: —— " —— $\theta / O\hat{y}$

Operateur selon vecteur unitaire, $\hat{u}$: $\vec{J}\cdot\hat{u}$

$\vec{J}\cdot\hat{u} = J_x \sin\theta\cos\varphi + J_y \sin\theta\sin\varphi + J_z\cos\theta$

Rotation d'angle $\alpha / O\hat{u}$: $R(\alpha)_{\hat{u}} = \exp\left\{-\frac{i}{\hbar}\alpha\,\vec{J}\cdot\hat{u}\right\}$

C'est plus facile d'étudier un rotation α / Oz

Stratégie : 1: rotation de $|jm\rangle$ de $-\varphi / Oz$

2: —— " —— $-\theta / Oy$

3. —— " —— α / Oz

4: —— —— θ / Oy

5: —— " —— φ / Oz

$\Rightarrow R(\alpha)_{\hat{u}} = R(\varphi)_z R(\theta)_y R(\alpha)_z R(-\theta)_y R(-\varphi)_z =$

$= \exp\left\{-\frac{i}{\hbar}\varphi J_z\right\}\exp\left\{-\frac{i}{\hbar}\theta J_y\right\}\exp\left\{-\frac{i}{\hbar}\alpha J_z\right\}\exp\left\{\frac{i}{\hbar}\theta J_y\right\}\exp\left\{\frac{i}{\hbar}\varphi J_z\right\}$

Je prends $R(\alpha)_{\hat{u}}\, U(\hat{n})|jm\rangle = R(\alpha)_{\hat{u}} R(\varphi)_z R(\theta)_y |jm\rangle =$

$= R(\varphi)_z R(\theta)_y R(\alpha)_z \underbrace{\exp\left\{\frac{i}{\hbar}\theta J_y\right\}\overbrace{\exp\left\{\frac{i}{\hbar}\varphi J_z\right\}\exp\left\{-\frac{i}{\hbar}\varphi J_z\right\}}^{=1}\exp\left\{-\frac{i}{\hbar}\theta J_y\right\}}_{=1}|jm\rangle =$

$= R(\varphi)_z R(\theta)_y \exp\left\{-\frac{i}{\hbar}\alpha J_z\right\}|jm\rangle$

J_z est diagonal dans la base $|jm\rangle$

$J_z|jm\rangle = m|jm\rangle$; $\exp\begin{Bmatrix} x & 0 \\ 0 & x \end{Bmatrix} = \begin{pmatrix} e^x & 0 \\ 0 & e^x \end{pmatrix}$

$$R(\alpha)_{\hat{u}}\, U(R)\, |jm\rangle = R(\varphi)_z R(\theta)_y\, e^{-i\alpha \frac{m}{\hbar}} |jm\rangle = e^{-i\alpha \frac{m}{\hbar}}\, U(R)\, |jm\rangle$$

$$\exp\left\{-\frac{i}{\hbar}\alpha\, \underline{\vec{J}\cdot\hat{u}}\right\} U(R)\, |jm\rangle = \exp\left\{-\frac{i}{\hbar}\alpha\, \underline{m}\right\} U(R)\, |jm\rangle$$

$$\Rightarrow \vec{J}\cdot\hat{u}\;\; U(R)\,|jm\rangle = m\;\; U(R)\,|jm\rangle$$

cqfd