

# Transformées de Fourier

## Définitions

### TF Directe

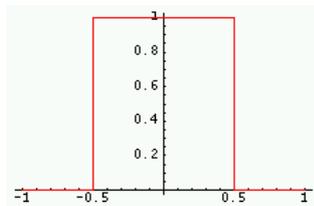
$$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(t) e^{-2i\pi\nu t} dt$$

### TF Inverse

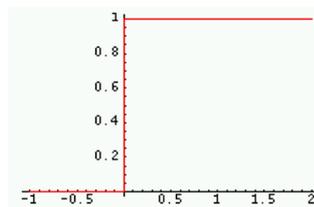
$$f(t) = \int_{-\infty}^{\infty} \hat{f}(\nu) e^{2i\pi\nu t} d\nu$$

### Divers

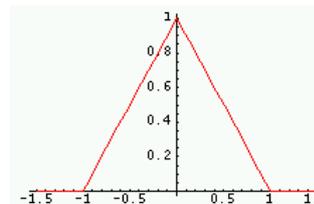
- Fonction porte :  $\Pi(t) = \begin{cases} 1 & \text{si } |t| < 1/2 \\ 0 & \text{si } |t| > 1/2 \end{cases}$



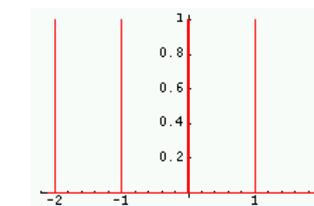
- Fonction de Heaviside :  $H(t) = \begin{cases} 1 & \text{si } t > 0 \\ 0 & \text{si } t < 0 \end{cases}$



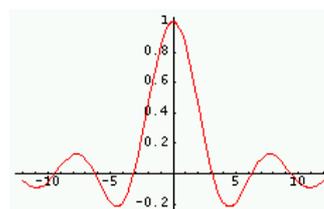
- Fonction triangle :  $\Lambda(t) = \begin{cases} 1 - |t| & \text{si } |t| \leq 1 \\ 0 & \text{sinon} \end{cases}$



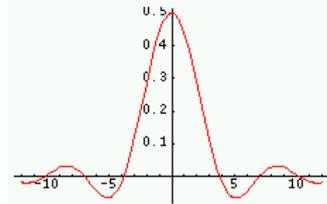
- Peigne de Dirac :  $\text{III}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n)$



- Sinus cardinal :  $\text{sinc}(t) = \frac{\sin t}{t}$



- $J_1$  cardinal :  $J_{1c}(t) = \frac{J_1(t)}{t}$



## Propriétés générales

Fonction	Transformée	Fonction	Transformée
$a f(t) + b g(t)$	$a \hat{f}(\nu) + b \hat{g}(\nu)$	$\frac{df}{dt}$	$2i\pi\nu \hat{f}(\nu)$
$f\left(\frac{t}{a}\right)$	$ a  \hat{f}(a\nu)$	$t \cdot f(t)$	$-\frac{1}{2i\pi} \frac{d\hat{f}}{d\nu}$
$\bar{f}(t)$	$\overline{\hat{f}(-\nu)}$	$f(t) \cdot g(t)$	$(\hat{f} * \hat{g})(\nu)$
$f(t + \tau)$	$\hat{f}(\nu) e^{2i\pi\nu\tau}$ <a href="#">Animation (340 KB)</a>	$(f * g)(t)$	$\hat{f}(\nu) \cdot \hat{g}(\nu)$
$e^{2i\pi\nu_0 t} f(t)$	$\hat{f}(\nu - \nu_0)$	$h(t) = \int \overline{f(\tau)} g(t + \tau) d\tau$	$\hat{h}(\nu) = \overline{\hat{f}(\nu)} \hat{g}(\nu)$

$$\hat{f}(0) = \int_{-\infty}^{\infty} f(t) dt$$

$$f(0) = \int_{-\infty}^{\infty} \hat{f}(\nu) d\nu$$

$$\hat{\hat{f}}(t) = f(-t)$$

Th. de Parseval

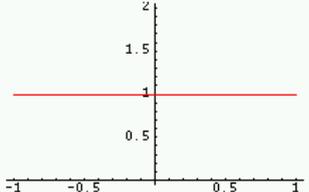
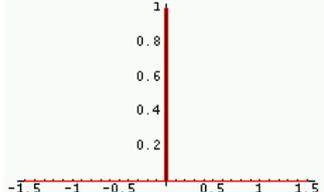
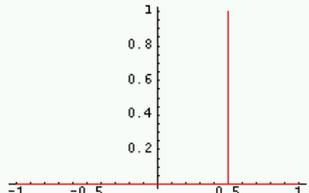
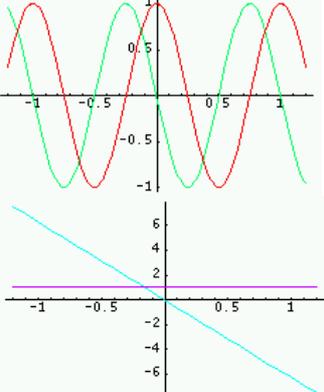
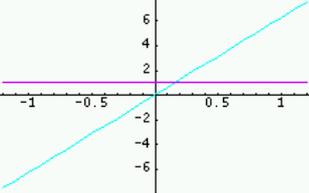
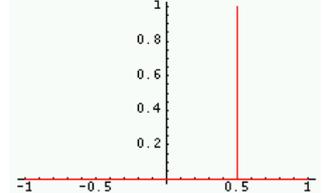
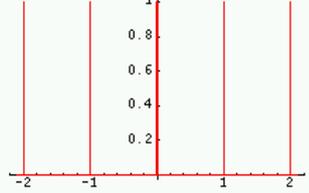
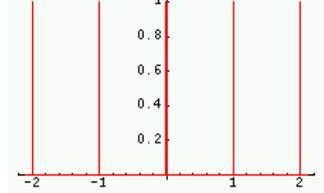
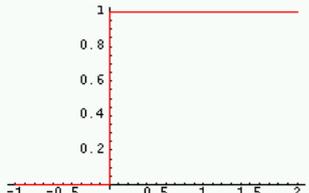
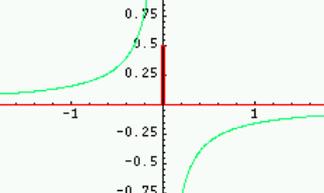
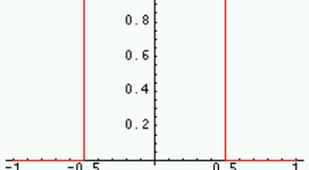
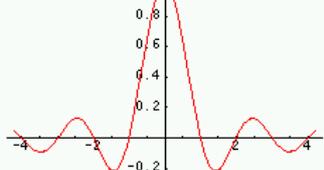
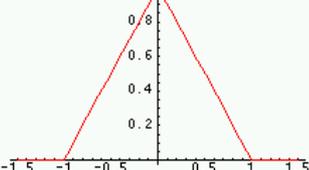
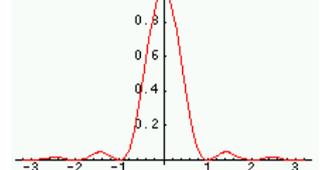
$$\int_{-\infty}^{\infty} f(t) \overline{g(t)} dt = \int_{-\infty}^{\infty} \hat{f}(\nu) \overline{\hat{g}(\nu)} d\nu$$

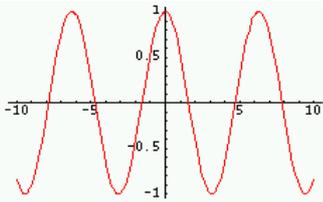
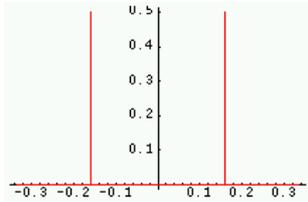
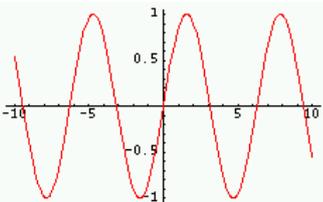
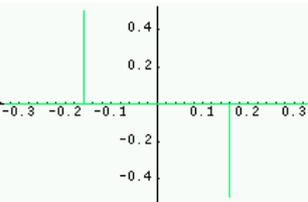
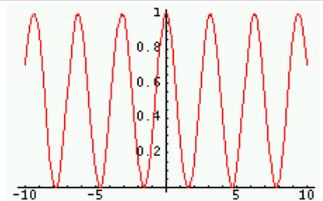
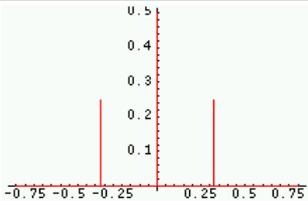
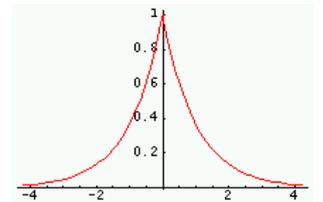
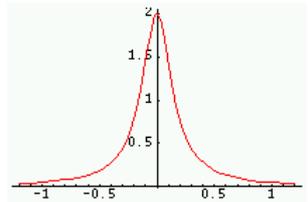
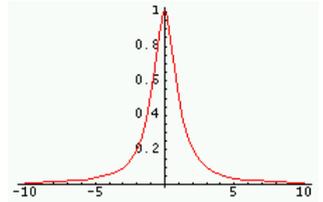
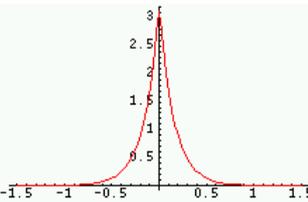
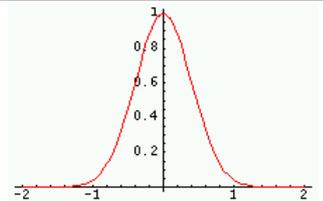
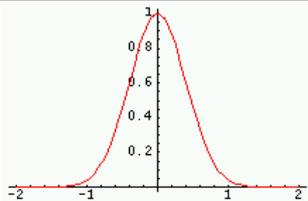
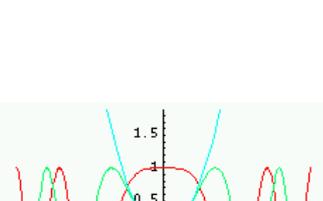
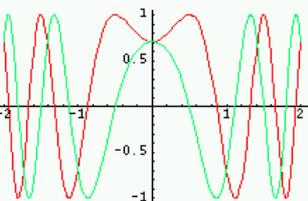
$f$  réelle paire  $\iff \hat{f}$  réelle paire     $f$  réelle impaire  $\iff \hat{f}$  imaginaire impaire

$f$  réelle  $\iff \hat{f}(-\nu) = \overline{\hat{f}(\nu)}$  (partie réelle paire, partie imaginaire impaire)

## Distributions à une dimension

Fonction	Graphe de la fonction <i>Rouge : partie réelle ; Vert : imaginaire ; Violet : module ; Cyan : phase</i>	Transformée	Graphe de la TF (parties réelle/imag ; module/phase)
$\delta(t)$		1	

1		$\delta(\nu)$	
$\delta(t - \tau)$		$\exp(-2i\pi\nu\tau)$	
$\exp(2i\pi m t)$		$\delta(\nu - m)$	
$\text{III}(t)$		$\text{III}(\nu)$	
$H(t)$		$\frac{1}{2}\delta(\nu) + \text{VP}\left(\frac{1}{2i\pi\nu}\right)$	
$\Pi(t)$		$\text{sinc}(\pi\nu)$	
$\Lambda(t)$		$\text{sinc}^2(\pi\nu)$	

$\cos(2\pi m t)$		$\frac{1}{2}\delta(\nu - m) + \frac{1}{2}\delta(\nu + m)$	
$\sin(2\pi m t)$		$-\frac{i}{2}\delta(\nu - m) + \frac{i}{2}\delta(\nu + m)$	
$\cos^2(\pi m t)$		$\frac{1}{2}\delta(\nu) + \frac{1}{4}\delta(\nu - m) + \frac{1}{4}\delta(\nu + m)$	
$\exp(- t )$		$\frac{2}{1 + 4\pi^2\nu^2}$	
$\frac{1}{1 + t^2}$		$\pi \exp(-2\pi \nu )$	
$\exp(-\pi t^2)$		$\exp(-\pi\nu^2)$	
$\exp(i\pi t^2)$		$e^{i\pi/4} \exp(-i\pi\nu^2)$	

## Distributions à deux dimensions

$$\hat{f}(u, v) = \iint_{-\infty}^{\infty} f(x, y) e^{-2i\pi(ux+vy)} dx dy$$

### TF Inverse

$$f(x, y) = \iint_{-\infty}^{\infty} \hat{f}(u, v) e^{2i\pi(ux+vy)} du dv$$

### Fonctions à variables séparables

$$f(x, y) = g(x) h(y) \iff \hat{f}(u, v) = \hat{g}(u) \hat{h}(v)$$

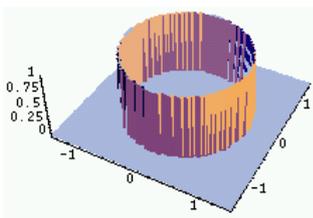
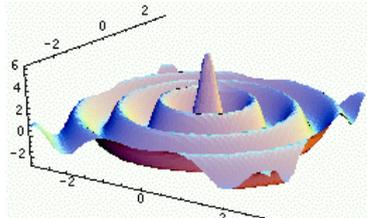
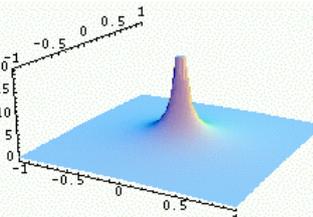
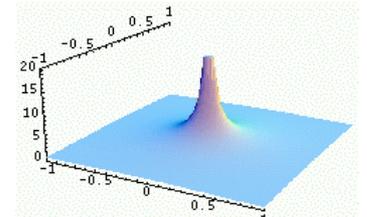
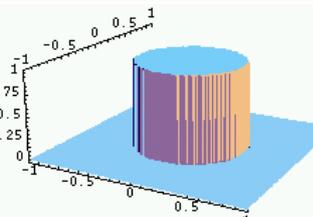
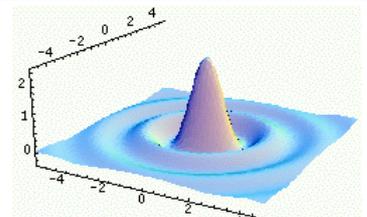
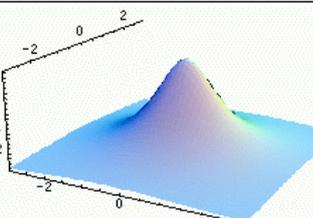
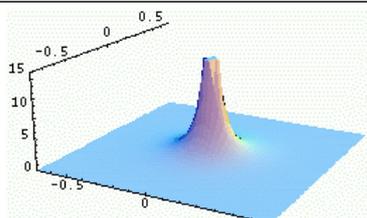
### Fonctions à symétrie de révolution

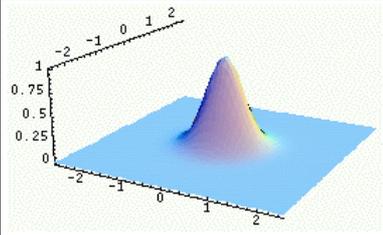
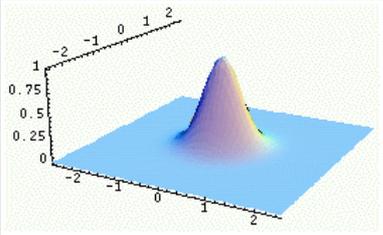
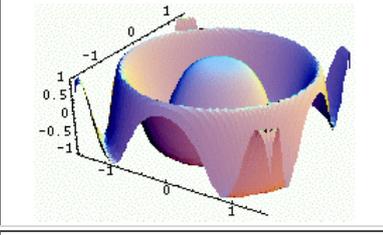
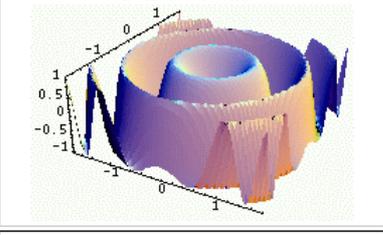
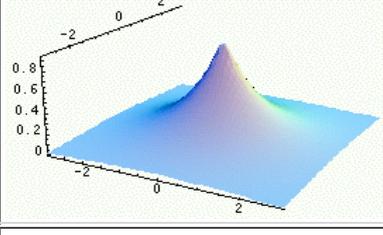
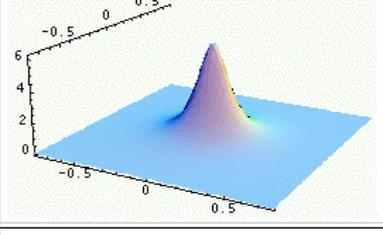
Soit  $r = \sqrt{x^2 + y^2}$  et  $q = \sqrt{u^2 + v^2}$ . Alors les TF directes et inverses bidimensionnelles sont aussi à symétrie de révolution et s'écrivent à l'aide de la transformée de Hankel

$$\hat{f}(q) = \int_0^{\infty} f(r) J_0(2\pi qr) 2\pi r dr$$

et

$$f(r) = \int_0^{\infty} \hat{f}(q) J_0(2\pi qr) 2\pi q dq$$

Fonction	Graphes de la fonction	Transformée	Graphes de la TF
$\delta(r - a)$		$2\pi a J_0(2\pi a q)$	
$1/r$		$1/q$	
$\Pi\left(\frac{r}{d}\right)$		$2 \left(\frac{\pi d^2}{4}\right) J_1(\pi d q)$	
$\frac{1}{\sqrt{r^2 + a^2}}$		$\frac{\exp(-2\pi a q)}{q}$	

$\exp(-\pi r^2)$		$\exp(-\pi q^2)$	
$\exp\left(i\pi\frac{r^2}{a^2}\right)$	Partie réelle 	$ia^2 \exp(-i\pi a^2 q^2)$	Partie réelle 
$\exp(-ar)$		$\frac{2\pi a}{(4\pi^2 q^2 + a^2)^{3/2}}$	
$r^2 f(r)$		$\nabla^2 \hat{f}$	